the changes produced, and the diseases to be guarded against in the subtle preparation of these different articles of daily use that the author is concerned.

The treatment of this varied assortment of subjects is somewhat condensed, no doubt from limitations of space, but the reader is supplied with a considerable amount of information about each, much of which is extremely interesting. In every case a useful outline of the method of manufacture or preparation is given, at all events as regards that stage of it in which fermentation is involved.

The treatment of the various processes dependent on alcoholic fermentation is too condensed to be entirely satisfactory, although all the essential points are touched upon. A somewhat disproportionate amount of space is devoted to the discussion of the "bios" question, which, however interesting in itself, is not of supreme technical importance. The fact, first observed by Wildiers, that yeast when inoculated in small amount into a synthetic medium fails to grow, was explained by him as due to the absence from such a medium of some specific material essential for yeast growth, and to this unknown substance he gave the name of bios. It is probable that the explanation advanced by Pringsheim, that yeast can only gradually adapt itself to the assimilation of the nitrogen of such media, is correct, although it seems also to be true that different yeasts possess very varying degrees of adaptability. In view of these facts it is remarkable that Pasteur was so successful in his classical experiments on the growth of yeast in simple media, and it has been suggested that this success was due to the presence of mycoderma in his yeast, the presence of this organism having been found to enable yeast to grow freely in media such as he employed.

The function of fermentation processes in the preparation of alcoholic beverages and vinegar is, of course, the fundamental one of producing the essential constituent—the alcohol or acetic acid—by biochemical change from the materials present in the liquid employed. When we turn to the other substances on our list, however, this is found to be by no means the case. Coffee consists of seeds or beans which occur firmly embedded in an integument, the whole forming the fruit of the plant. The chief function of the fermentation in this case seems to be the loosening of this integument, so that the beans can readily be separated and dried, and this is effected by the decomposition of a viscid layer immediately surrounding the seeds, an alcoholic fermentation of this material occurring first, and being followed by a stage in which acetic acid is produced, a considerable rise of temperature accompanying the change.

Black tea and tobacco, on the other hand, are submitted to processes of fermentation the object of which is the attainment of the flavour and aroma upon which their value depends. In both cases opinions are divided as to the exact nature of the process. On one hand it is maintained that micro-organisms are essential agents and it has even been proposed to impart the aroma of the finer qualities of tobacco to inferior material by inoculation with the appropriate organisms. On the other hand it is contended that the change is enzymic, due to the decomposition of glucosides and to oxidation processes, although it is admitted that organisms are present and may have some secondary and minor effect on the result. In the case of cocoa, too, some doubt exists as to the relative share of plant enzymes and extraneous organisms in the fermentation to which the beans are submitted, and which results in a dry bean of good flavour and colour.

The preparation of mustard has been specially investigated by the author. A mixture of ground white and black mustard seeds (respectively containing sinalbin and sinigrin) is treated with 2·5 per cent. acetic acid, along with salt and a mixture of spices. The chief change which occurs is the decomposition of the glucosides with the liberation of the mustard oils. The mass is then left for a few days, ground and bottled. Since the glucosides of mustard are remarkably resistant to the attack of most bacteria and the mustard oils are strong inhibitants of bacterial growth, although not particularly powerful antiseptics, it might be thought that mustard would be free from liability to bacterial "disease." This is, however, not the case, and the author has isolated two species of sporing bacteria (*B. sinapivorax* and *B. sinapivagus*) which are capable of decomposing the mustard glucosides, in one case with evolution of gas, and occasionally cause serious loss to the manufacturer. The book, which, as will be seen, deals with an extremely interesting subject, is provided with a good index and bibliography, and is adequately illustrated.

A. HARDEN.

SOME RECENT WORKS ON MATHEMATICS.

(1) *Lehrbuch der Mathematik für Studierende der Naturwissenschaften und der Technik: Einführung in die Differential- und Integralrechnung und in die analytische Geometrie.* By Prof. G. Scheffers. Zweite Auflage. Pp. viii+732. (Leipzig: Veit and Co., 1911.) Price 18 marks.

(2) *Die Integralgleichungen und ihre Anwendungen in der mathematischen Physik.* By Adolf Kneser. Pp. viii+243. (Braunschweig: Fr. Vieweg und Sohn, 1911.) Price 6 marks.

(3) *Untersuchungen über Oszillationstheoreme.* By Dr. phil. Otto Haupt. Pp. 50. (Leipzig and Berlin: B. G. Teubner, 1911.) Price 2 marks.

(4) *Die partiellen Differentialgleichungen der mathematischen Physik, nach Riemann's Vorlesungen in fünfter Auflage bearbeitet.* By Prof. Heinrich Weber. Erster Band. Pp. xviii+528. Zweiter Band. Pp. xiv+575. (Braunschweig: F. Vieweg und Sohn, 1910, 1912.) Price 12 and 15 marks.

(5) *The Dynamical Theory of Sound.* By Prof. Horace Lamb, F.R.S. Pp. viii+303. (London: Edward Arnold, 1910.) Price 12*s.* 6*d.* net.

(6) *A Logical Notation for Mathematics.* By Robert T. A. Innes. Pp. 3. (Cape Town: S.A. Association for the Advancement of Science, 1911.)

(7) *Vorlesungen über Variationsrechnung.* By Prof. Oskar Bolza. In drei Lieferungen. Pp. iv+iv+x+706+10. (Leipzig and Berlin: B. G. Teubner 1908–9.) Price 19 marks.

(8) *Die Prinzipien der Mechanik für eine oder mehrere von den räumlichen Koordinaten und der Zeit abhängige Variabeln, II.* By Leo Konigsberger. Pp. 24. (Heidelberg: Carl Winter, 1911.)

(9) *Theoretische Mechanik.* By Prof. R. Marcolongo. Autorisierte deutsche Bearbeitung. By Prof. H. E. Timerding. Erster Band, Kinematik und Statik. Pp. viii+346. (Leipzig and Berlin: B. G. Teubner, 1911.) Price 10 marks.

(10) *Sur la notion de Courbure, et sur quelques points de Geométrie infinitésimale non euclidienne.* By C. Cailler (Mémoires de la Société physique et d'Histoire naturelle de Genève xxxvii, 2.) Pp. 62. (Genève: Georg et Cie., 1911.) Price 5 francs.

(11) *Proceedings of the London Mathematical Society.* Second series. Vol. ix. Pp. xvi+489. (London: Francis Hodgson, 1911.)

(12) *Bulletin of the Calcutta Mathematical Society.* Vol. i., No. 3 (October, 1909). Pp. 70. (Calcutta: Mathematical Society, Senate House, Calcutta, 1911.) Price 10 rupees per year.

(13) *Revista de la Sociedad matemática española,* 1–5 (Mayo–Diciembre, 1911). Pp. 40–76. (Madrid: Dr. José Nungot, Universidad Central, 1911.)

ONE of the most important facts which modern mathematicians now realise is that the principles of the differential and integral calculus can be taught to beginners by simple methods involving only a knowledge of the rudiments of algebra, and later on of trigonometry. There are not many teachers still in the dark on these points, though a remarkable exception occurred recently when an anonymous author wrote a book intended to show that certain fools called mathematicians had made the calculus unnecessarily hard, and proved his point, not in the way he probably contemplated, but by establishing the fundamental formulæ with a wasteful luxuriance of infinite series and disregard of small quantities which would have formed a more fitting subject for a book entitled "Calculus Made Difficult." The change has been marked in England by the appearance of a flood of school calculuses, reminding one of the former flood of school geometries.

(1) In these circumstances English teachers will derive considerable interest from studying the elementary but rigorous treatment of the subject in Dr. Scheffer's "Lehrbuch der Mathematik." The first chapter introduces the arc or radian measure of angles, and deals with functions in general. It opens with a careful comparison of the uses of graphic and analytical methods of solution, which should be a lesson to those teachers who try to hide up the shortcomings of constructive geometry by making their pupils use hard pencils. The notion of a differential coefficient is introduced in the first instance by consideration of linear and quadratic functions. Then follow the formulæ for algebraic functions deduced from the sum and product rules, and the rule for functions of functions, which is here enunciated as the chain rule. Differentiation of a power is deduced from the product rule by mathematical induction, as it should be, the use of the binomial theorem being avoided. The elements of the integral calculus are treated next, and it is only after this that the notion of a natural logarithm is shown to follow directly from the calculus, exponential functions being taken in the succeeding chapter, and trigonometric functions coming next. The chapter dealing with these contains a synopsis of trigonometry. For English readers the hyperbolic notation ("ſin, cos," printed in German type) is perhaps inconvenient. Then follow successive differentiation, maxima and minima, curvature and evolutes, particle dynamics, Lagrange's and Taylor's formulæ, miscellaneous methods of integration, Fourier's series, and partial differentiation. At the end is a useful collection of tables and integration formulæ. A large number of applications are given in the form of examples, some completely worked out, others left to the reader.

It need scarcely be pointed out that no two writers would agree as to what should be included in a book of this kind, and what should be omitted. On the whole, this book tends on the side of thoroughness, rigorous development, and careful discussion of points of detail, notably in dealing with such matters as continuity. Possibly there may be few students of physics in this country who do not have to skip over and take for granted some of the arguments. But in such matters as order of treatment and rendering the subject independent of an extensive previous knowledge of algebra and geometry, the book pretty nearly reaches the goal towards which modern teachers have been striving.

The influence of pure mathematics on the progress of mathematical physics is well shown by the next group of books under review. The partial differential equations of physical problems have received so much attention at the hands of both mathematicians and physicists that we had begun to think that their study had reached a stage of finality in which nothing further of importance remained to be done. But during the last ten years harmonic analysis has been completely revolutionised by the development of the theory of integral equations, which places a new and powerful weapon in the hands of the applied mathematician. This modern theory traces its origin back to Fredholm's paper of 1900 on Dirichlet's problem, and the most important subsequent works are those due to Stekloff, Hilbert, and Schmidt; in particular Hilbert's "Foundations of a General Theory of Linear Integral Equations."

(2) Dr. Kneser's contribution to the new subject is exactly described by the following notice of the publishers:—

> "The present work develops the theory of integral equations not from the starting point of analytical generalities but from the theory of heat-conduction, of free and forced oscillations and of the potential. The author thus hopes to meet the requirements of those mathematicians and physicists who wish to apply the new analytical method to concrete questions."

Now, in endeavouring to get the whole hang of the problem, so to speak, condensed in a nutshell, or, in other words, to find out what the investigations are driving at, and to put the matter in a form in which it could be explained to a pupil in ten minutes, the present reviewer encountered a slight difficulty at the outset.

Dr. Kneser starts with the problem of linear conduction of heat, but the reduction to an integral equation depends essentially on the introduction of Green's function. Now we have always regarded Green's function as connected with the problem of the potential; and, further, most elementary English books on analytical statics get no further than explaining Green's *theorem*, and possibly stating Green's *problem*, but not defining Green's *function*. The necessary light was thrown on the subject by referring to p. 241 of Weber's treatise reviewed below, which not only gives, for the problem of the potential, the definition of Green's function, but shows, further, how this leads at once to an *integral equation* for the potential of a given distribution subject to given boundary conditions. In the case of the potential due to *fixed* charges, this integral equation, of course, degenerates into an ordinary integration formula.

In heat conduction and allied problems Green's function between two points, P and Q, might be defined as the temperature or potential function at one point due to a unit source, at the other subject to the given boundary conditions. This function is the nucleus or kernel (Kern) of the integral equation, *i.e.* the factor which multiplies the unknown variable under the sign of integration.

After heat conduction in one dimension, Dr. Kneser applies the method to the problem of small oscillations, Sturm-Liouville's functions, and problems in two or three dimensions, the remaining sections being devoted to the existence theorem, Dirichlet's problem, and Fredholm's series.

(3) A closely allied line of investigation is developed in Dr. Haupt's pamphlet on "Oscillation-theorems," which is divided into two parts, the first dealing with the general linear homogeneous differential equation of the second order containing an arbitrary parameter in its undifferentiated coefficient, while the second deals with the special differential equation of the fourth order, in which the second differential coefficient of a multiple of the second differential coefficient of the dependent variable is proportional to a multiple of the variable itself, the multipliers being functions of the independent variable. It is largely a development of Hilbert's work on integral equations, and deals in particular largely with the conditions under which such differential equations may lead to a "Green's system," and the corresponding forms of the boundary conditions.

(4) That integral equations are destined to play an important part in the formal treatment of mathematical physics is now evident. It does not appear, however, probable that they will at present supersede the use of harmonic analysis for purposes of calculation. In bringing out a fifth edition of his "Partial Differential Equations based on Riemann's Lectures," Prof. Weber plainly states at the outset that he has not been able to rewrite the book on the lines of recent researches, partly owing to want of time and energy, and also partly on the ground that the subject is now in a transient stage, in which further developments may be expected every day, so that if an attempt were made to start afresh, the whole book would soon be out of date. In addition to the theory of integral equations, a second line of recent development has grown up in the study of the principle of relativity.

In these circumstances Prof. Weber has adopted the most desirable course, namely, to introduce references to this recent work into the text at suitable places, and his book still constitutes as good an introduction to the study of mathematical physics as could well be written. Although the book has more and more become the work of Weber himself, he still desires to pepetuate the name of Riemann as having sown the seed from which this large tree has grown up.

The second volume has only just appeared. It deals with the theory of certain differential equations, heat-conduction, elasticity, theory of vibrations of strings, and membranes, electric oscillations, and hydrodynamics.

We have referred to the unsuitability, for English readers, of the German hyperbolic notation, but a greater difficulty is introduced into these German treatises by the use of a single integral sign to denote surface and volume integrals. It certainly adds considerably to clearness of exposition to use double and triple integral signs in these cases. Of course, this method is illogical as practised in this country, where a triple sign of integration is often followed by a single differential. The correct plan in such cases is to denote surface and volume differentials by d^2S and d^3V instead of dS and dV, and it is much to be wished that this were always done.

(5) The somewhat brief discussion of vibrations in the last-named treatise is a reminder that, unfortunately, a review of Prof. Horace Lamb's "Dynamical Theory of Sound" has been unavoidably delayed for an inordinate time. The subject is one which lends itself to treatment in three ways; by the publication of memoirs on specialised researches, by the production of a treatise even larger and more exhaustive than Lord Rayleigh's three volumes, and by the compilation of an introductory treatise in which the most important fundamental principles are dealt with concisely, over-elaboration being avoided. Prof. Lamb has chosen the last alternative, and has thus produced a book which should be of great use to students of applied mathematics, physics, and acoustics.

This aim at brevity necessitates the omission of details of long analytical investigations, the results of which are stated without proof; for example, under Fourier's series no attempt is made to reproduce the existing literature relating to its convergency, and the theory of vibrating systems in general is discussed briefly without reference to more than a statement of results in connection with transformation to normal coordinates. The various chapters following the introduction deal with general theories of vibration, strings, Fourier's theorem, bars, membranes and plates, plane and spherical sound waves, generation and diffraction of waves, pipes and resonators, and physiological acoustics, the last-named chapter being a summary of a branch of acoustics falling outside the general scope of the book. It is probable that the average student would do better to follow a course of this kind, and then single out some subject for specialised study rather than to spend time on reproducing analytical investigations about the truth of which no doubt exists.

We notice that both Weber and Lamb assume that in a membrane the stress is the same in all directions—in other words, isotropic. Is this necessary? Analytically speaking, if the stress is homogeneous but not isotropic the membrane can be projected orthogonally into one in isotropic stress; on the other hand, it is fairly certain that in an actual membrane, such as that of a concert drum, it is very difficult to adjust the tension round the boundary so as to make the stresses either homogeneous or isotropic. A material membrane probably differs in its physical properties from the ideal membrane (Lamb, p. 189) to the same extent that the substance of which it is composed differs from a fluid. If this view is not correct further discussion is needed.

(6) Reference to notation under (4) leads us to Mr. Robert Innes's short Cape Town note, in which he recommends the use of small letters, Roman, italic, Clarendon, and Greek for symbols of quantity, and capital letters for symbols of operations, such as S for sine, D for differential coefficient. He does not tell us what he would do with separate differentials. In view of the fact that after these many years we still have to write cosec x for what is logically $\sin^{-1}x$, and that a Frenchman or German cannot *say* such a number as 394 in the form in which it is written, it is not much use suggesting reforms, except in notation the use of which is very limited.

(7) In a review like the present, it would be impossible to enter into a detailed criticism of the German edition of Dr. Oskar Bolza's "Lectures on the Calculus of Variations." While based on his American work bearing the same title, these three volumes aim at a more comprehensive treatment of the subject; at the same time, the author does not claim to have exhausted the theory even in the seven hundred pages which he has devoted to it. What he has rather aimed at has been to give a fairly elementary outline of the main principles of the calculus of variations, together with a more detailed treatment of its modern developments. The author's claim to have clearly expounded the fundamental definitions and methods, with the assistance of suitable geometrical and other illustrations, is fully justified by a survey of the contents. At present there are only a few people in England who study calculus of variations, and it is to be hoped that the subject will become more popular in the future in view of its important applications to physical problems. "Differentiation of an integral," which leads directly to "variation of an integral," is really one of the easiest things in the calculus to treat in an elementary way, and there can be no excuse for keeping this study, so to speak, under lock and key, only to be shown to students on rare occasions. The student of physics who goes no further than the first chapter of this book will be able to obtain a proof of Lagrange's equations of motion, together with a knowledge of the principle of least action, which will do him far more good than letting x equal a function of t, θ, and ϕ, and writing out by heart a proof of these equations involving some juggling with differential coefficients the physical interpretation of which is not obvious.

On the other hand, the need for an exposition of modern developments of the subject is shown by the many recent papers that have appeared. For example, a paper in German on the invariant form of the second variation of a double integral is contributed to the Proceedings of the Tokyo Mathematical and Physical Society for September, 1911, by M. Fujiwara.

(8) Closely allied, as leading to a generalised form of the principle of least action, are Prof. Konigsberger's papers on the principles of mechanics for one or more dependent variables, of which the second part has reached us.

(9) An examination of the contents of "Theoretische Mechanik" leads to the impression that Dr. Timerding showed want of judgment in undertaking the translation into German of a book like that of Marcolongo's. It is known that Profs. Marcolongo and Burali Forti have been engaged in drawing up a report on vector notations, as to the value of which considerable differences of opinion exist. We should not, however, take any exception to the book on this ground; but when we find in the second chapter difficult theorems in potential analysis, such as Stokes's and Green's theorems, and Poinsot's construction for the motion of a rigid body under no forces in chapter vii., while it is not until much later on that the author deals with such elementary notions as the parallelogram of forces, the principle of the lever, equilibrium on an inclined plane, and the principle of Archimedes, the book may fairly be regarded as affording a lesson as to how mechanics should *not* be taught. It forms a striking contrast to the clear and practical exposition of advanced dynamics contained in Webster's book, published by the same firm. It is becoming more and more recognised every day that the study of mechanics should be approached by beginners from the experimental side, and for this purpose elementary statics and hydrostatics form the best starting point. With such a preparation, a pupil may be able in time to grasp the nature and use of vector analysis, but to start him, as this book does, at the wrong end would be fatal.

(10) In his paper on curvature, Prof. Cailler has set before himself the task of formulating a theory of curvature of a sufficiently comprehensive character to cover the two cases of Riemann's and Lobatschewski's non-Euclidean geometries. To do this he has found it necessary to start with an algebraic definition of curvature based on considerations of kinematic geometry, and while admitting that some of the notions in his paper are of a somewhat abstract character, Prof. Cailler claims that the nature of the problem and the differences existing between the various kinds of space to which it refers render such a treatment necessary if it is to possess the required degree of generality.

(11) The volume of the London Proceedings before us continues to afford evidence of the good work that

is being done by those English mathematicians who, by denying themselves the luxuries of life, or otherwise, are able to give time and thought to original work. Applied mathematics is only represented by Dr. Nicholson's paper on scattering of light by a conducting sphere, and Mr. F. B. Pidduck's note on stability of rotating shafts. The most substantial contribution to pure mathematics is Dr. W. H. Young's series of seven papers dealing with various points regarding the foundations of the differential and integral calculus. At a time when the teaching of this subject to elementary classes is receiving so much attention, it is most fortunate that our country has been able to produce a mathematician like Dr. Young, who is underpinning this structure with a foundation of rigorous reasoning. For example, one paper is called "A Note on the Property of Being a Differential Coefficient," while in another paper on "A New Method in the Theory of Integration," the author proves a number of interesting results, such as that "an upper-upper-lower-upper (lower-lower-upper-lower) semi-continuous function is an upper-lower-upper (lower-upper-lower) semi-continuous function."

Lieut.-Colonel Allan Cunningham writes "An 8-vic, 16-ic, . . . , Residuacity," while it may be desirable to explain that Mr. G. T. Bennett's note on "The Double Six" refers to certain lines associated with quadric and cubic surfaces. We have one fault to find with the binding; the top edges of the pages are cut, but the outside edges are left rough, and are very uneven, consequently the pages soon get to look untidy; and, furthermore, the book always opens at certain pages and never can be opened at others.

One cannot, however, help regretting that there are probably many hundreds of persons in this country engaged in teaching mathematics who are ignorant of the very existence of these Proceedings. It would be a great advantage in this respect if the London Mathematical Society and the Mathematical Association could be amalgamated so that the educational and "original work" aspects of mathematics could be brought into closer touch with each other, as appears to be done in the American Mathematical Society. A very similar fusion of theoretical and practical interests has just been effected in the Aëronautical Society.

(12) One thing further is needed, namely, the publication of a summary of current mathematical literature, with abstracts of the principal papers. This is furnished by the Bulletin of the Calcutta Mathematical Society, which, in addition to original papers by Cullis and Mukhopadhyaya, contains a summary of the principal mathematical journals, reviews, lists of papers under "Societies and Academies," "Notes and News," "New Publications," and an obituary notice of Prof. Simon Newcomb.

Assuming that the Calcutta society is sufficiently powerful efficiently to continue the publication of these notes and abstracts, it is surely somewhat humiliating to think that the mathematicians of a Western civilisation should have to send to Calcutta for a list of the papers which they themselves have written. But as it is stated that the Bulletin will appear four times a year, while the number before us for October, 1909, bears the date of printing 1911, and was only received for review on October 14 last, we have some fears lest the task which these Calcutta mathematicians have undertaken may have proved too heavy for them. If so, we hope that European mathematicians will take steps to ensure the continuation of a chronicle which is unique of its kind.

(13) Although it has been mentioned in the "Notes" column of a previous issue of NATURE, the Spanish society's "Revista" may with advantage be referred to in this connection as showing that Spain has obtained an amalgamation between theory and practice of the kind which we in England have hitherto failed to attain. On the theoretical side we have papers on such questions as summation of series, polygonal numbers, generalisation of the nine-point circle, meridian arcs, and equilibrium of a moving chain (the figures, however, show the rubber belt of the barber's hair-brushing machine upside down, and the wheel moving in the opposite sense to the belt). On the more popular side we have biographies of Fermat, Nunez, Benaflah, and Siliceo, a history of Spanish mathematics, reviews, notes and news, and notes and queries columns. In addition, the society is drawing up a vocabulary of Spanish mathematical terms. We cannot better close this review than by quoting an extract from the article, "Sursum corda," by Captain Durán Loriga, on the functions of the newly founded society, in which he says, "The first which imposes itself is to create a *mathematical atmosphere* (*ambiente matemático*), in particular to convince the whole world that without that great science it would be impossible to approach the study of natural and physical sciences, which daily tend more and more to assume a mathematical form."

This quotation is not the only or even the most powerful argument in Captain Loriga's article, which further refers to the epoch-making discoveries of Abel (who lived and died in great poverty), Kowalewski, and others, and their influence on the progress of applied science. But his notion of an "ambiente matemático" should be kept prominently in mind by all mathematicians who are able and willing to take part in the campaign which in our country requires to be waged against "England's neglect of mathematics." This campaign can only be successful if everyone who is interested in the progress of pure science lends a helping hand. If they will not, then we must let the coming generation make the best it can of the knowledge which it has inherited from its forerunners, and leave posterity to fend for itself.

G. H. BRYAN.

OUR BOOK SHELF.

The Principle of Individuality and Value. The Gifford Lectures for 1911, delivered in Edinburgh University, by Dr. B. Bosanquet. Pp. xxxvii+409. (London: Macmillan and Co., Ltd., 1912.) Price 10*s*. net.

THIS may be described as a reply to the critics of Absolutism, mainly, of course, the late Prof. James. Dr. Bosanquet is always as readable as the difficulty of his subject permits, and his rejoinders are always moderate and courteous. The reader feels that the